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# Measuring Flood Risk in Czechia with Stress Testing and a Gumbel copula based VaR

Marek Folprecht

## Abstract

The study presents a holistic approach to modeling flood risk of real estate properties. The method combines a hydrological flow simulation model and a model of financial losses. Two use-cases of the model are discussed. First, a stress testing method, based on historical scenario simulations, is presented. Next, a Value at Risk approach using the Generalized extreme value distribution and the Gumbel copula is discussed. Both methods are then tested on a large sample of Czech house data. The results show that the model can replicate the order of historical flood magnitudes under the historical scenarios. Moreover, the Value at Risk approach can generate scenarios unseen in recent history. The model could be a useful flood losses modeling tool for banks, insurance companies, real estate investment companies or state agencies. A special case for stressing credit risk parameters for mortgage portfolios is discussed in more detail.

**AMS/JEL classification:** C63, Q54, G32

**Keywords:** Flood risk, Generalized extreme value, Gumbel copula, Value at Risk, Monte Carlo, Czech Republic, Stress Testing

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## 1. Introduction

Inundation event is reported to be the most economically damaging type of natural disaster in the United States (Shu et al., 2022) and the most frequent natural disaster in the world (CRED, 2020). Growing evidence of climate change manifested in requirements to simulate climate related damage for financial institutions. In the last years following the Paris agreement and the Green deal, banking regulators, especially in Europe, require banks to simulate potential losses from climate-related scenarios. Such an example is the ECB climate stress test in 2022 (Germann et al., 2023; ECB, 2022). Climate scenarios are provided by institutions such as the Network for Greening the Financial System (NGFS, 2024) or developed by individual banks using their own methodology.

While the climate risk topic is relatively new in the financial risk management domain, the methodology is not settled and is often developed on an ad hoc basis tailored to a specific stress test exercise.

A financial stability impact of flood risk in the Netherlands was analysed by (Caloia & Jansen, 2021). Using official flood hazard maps, they measured the financial impact of floods at post-code level areas using a set of 6 scenarios. The authors have a strong assumption that all areas at risk are assumed to be affected simultaneously. The conclusion is that the Dutch banking sector has sufficient capital to withstand flood scenarios.

The ECB stress test (ECB, 2022) included a 1 year Europe-wide severe flood risk scenario. A flood risk map at the NUTS3 level is constructed by the regulator, mapping each region into one of four risk categories (minor, low, medium, high). Then, for each category, a price shock is specified (ranges between 4-45% for residential real estate prices). Banks should translate the price shock into LGD values. Centralized price shocks are not methodologically transparent and the purpose of the regulator stress test is to measure the vulnerability to flood risk rather than to simulate accurate asset-level floods given a probabilistic model with diversification assumption. Not all assets are flooded at the same time so assuming a shock of 4-45% is rather high as caused by floods directly, although spillover effects may affect even house prices of other houses. As ECB stress test is rather exploratory, it does not constitute capital requirements. Capital requirements are often based on probabilistic models such as VaR.

The purpose of this paper is to develop a holistic approach to modeling flood risk without the need of excessive subjective ad hoc assumptions and expert opinions. Assumptions like simultaneous floods in the whole country (or the area that the bank is most exposed to) and arbitrary flood severity are common in both regulator and internal stress tests. Such approach is a tool for vulnerability analysis but it cannot produce reliable flood risk measurements and scenario probabilities. The study addresses the question how the flood risk can be quantified so that the results are statistically robust and practically useful. The model presented in this article aims to be holistic as it can be used for multiple purposes such as capital requirement calculation, simulation of historical severe flood events, climate change simulation, assigning probability to hypothetical flood risk scenarios, calculating flood risk house price discount or forecasting future flood damage beyond the use for banking institutions (state funds, insurance companies etc.). It is also measured on asset level (unlike postcode or NUTS region level) so that it respects individual asset risk as well as the dependence between flood occurrences in different parts of the country (no assumption of simultaneous flood occurrence). Finally, the model is continuous (generates any number of flood scenarios given the probability model) and does not rely on a set of discrete set of scenarios.

As a benchmark and inspiration, the market risk modeling framework was used. Typically, market risk model consists of two components. First, it is the marginal risk factor loss distribution. In a simplified form, it can be viewed as the asset-level loss distribution. Second, the dependency structure. It can be the dependency captured in historical returns or correlation and copula models. The flood risk is approached the same way in this article. First, the marginal distribution of the flood risk losses is estimated (the *financial loss simulation* model) by calibrating the model to flood hazard maps. Combining multiple flood hazard maps for different return periods (such as 1-in-20 and 1-in-100 years) gives a marginal loss CDF on asset level. Second, the dependency structure is captured either by observed historical return periods at different places (stress test exercise) or by a Gumbel copula model (VaR model) fitted on a water flow data from hydrometeorological stations (the *hydrological flow simulation* model).

The rest of the paper is organized as follows. First, the financial loss simulation method is presented in II. Next, calibration of the model to available data is discussed, followed by a section on how to integrate the model with hydrological flow simulation model. Finally, model calibration to hydrological flow data is presented in III and empirical results of losses under the Stress Testing and Value at Risk paradigm are presented in IV, together with several backtests of the model assumptions. Advantages of the method and its limitations are discussed in V.

## II. Research Method

### Financial loss simulation

First, the approach to simulate financial losses given flood return period (such as 1-in-100 years) realization is discussed. The model generating financial losses is referred to as financial loss simulation model. Later, hydrological flow simulation model is introduced, which generates the input to this model, flood return periods.

### Financial Loss Distributions

Assume that we are given a water depth flood map for different return periods (such as 1-in-20 or 1-in-100 years). These are essentially functions that for each place (or region) on the map return the water depth conditional on the floods realizing.

These flood maps are not independent. If a house is flooded in a lower return period (e.g., 1-in-20), it is automatically flooded in higher return periods (e.g., 1-in-100) and the severity, measured by water depth, is higher in that higher return period flood.

So far, we have a list of water depths for each house in the dataset. It may look as follows:

House id	1-in-20 Flood Depth (m)	Year	1-in-100 Flood Depth (m)	Year	1-in-500 Flood Depth (m)	Year
H_01	0.3		0.8		1.3	
H_02	0.0		0.4		0.9	

**Table 1 Example of extracted flood depth data from flood maps**

Flood zone is the lowest return period for which a house is impacted by flood risk (water depth is positive). In the example above, house H\_01 is in the 1-in-20 years flood zone and house H\_02 is in the 1-in-100 years flood zone.

Note that we are only given quantiles of flood depth distribution for a set of discrete probability values. The 1-in-20 year flood depth corresponds to the 95th percentile of the water depth distribution, while the 1-in-100-year flood depth corresponds to the 99th percentile. Now the task is to fit a continuous model which can closely reproduce these quantiles. This is solved by fitting a Gumbel mixture distribution to the flood depth data.

The term mixture represents the fact that the flood depth distribution is divided into 2 subcases. First, there is a high probability,  $p_0$ , of flood depth being zero (no floods happening). Second, there is a probability of  $1 - p_0$  of flood depth being positive. So, with a probability of  $p_0$ , the first mixture component is realized. This is essentially a degenerate random variable with Dirac delta function probability density function. In other words, the flood depth is equal to zero with 100% probability if this mixture component is realized. Second, with a probability of  $1 - p_0$ , the second mixture component is realized. Then, a flood depth based on a draw from the Gumbel distribution is applied.

In this study, to ensure more robustness against outliers and incomplete location information, flood depth quantiles are aggregated for each flood zone by applying average. In other words, instead of fitting a loss distribution for each house separately, the loss distribution is fitted for an average house within a flood zone. This was done in two ways: first, across all points on the flood map (excluding water courses), and second, at the actual locations of houses within each flood zone. Since both approaches yielded similar results, the former was adopted, as it is not limited to house locations. The

resulting aggregation in Table 2 displays the average quantiles of water depth on flood zone level instead of individual house level as it is the case in Table 1.

Flood zone	1-in-20 Flood Depth (m)	Year	1-in-100 Flood Depth (m)	Year	1-in-500 Flood Depth (m)	Year
1-in-20	0.2		0.7		1.3	
1-in-100	0.0		0.5		0.9	

**Table 2 Example of aggregated flood depth data from flood maps**

Subsequently, for each flood zone (FZ), the water depth  $X$  is modeled using a Gumbel mixture distribution:

$$F_{mix}(X | FZ) = p_0(FZ) + (1 - p_0(FZ)) \cdot F_{Gumbel}(X, loc(FZ), scale(FZ)) \quad (1)$$

Where:  $F_{Gumbel}$  represents the CDF of Gumbel distribution,  $p_0(FZ)$  gives the probability of no floods,  $loc(FZ)$  and  $scale(FZ)$  denotes the location and scale of Gumbel distribution. Parameters are separately fitted for each flood zone (FZ).

To find the mixture parameters ( $p_0$ ,  $loc$  and  $scale$ ), the least squares method was used. The method finds parameters that minimize the squared distance between the observed average quantiles of water depth and the corresponding theoretical quantiles of the Gumbel mixture distribution. The resulting calibrated model gives us a separate water depth distribution for each flood zone.

The Gumbel distribution was chosen first because it is right tailed and therefore captures the fact that flood losses can be severe with low probability and second because it fitted the observed data points of CDF well. In Table 3, there is a comparison between three different choices for the second mixture component. First, the in-sample  $R^2$  is calculated and is near 1 for all three models. Second, the LOO (leave one out)  $R^2$  and MAE is computed using predicted residuals obtained from successively omitting each observation from the estimation sample. Fréchet and Gumbel mixture have a high predictive performance with LOO  $R^2$  around 90% while lognormal mixture has a medium predictive performance of 69%. Gumbel mixture has the lowest MAE while Fréchet mixture has the highest. Overall, Gumbel mixture was chosen as it has the best MAE and second best  $R^2$  out of sample performance.

	Gumbel mixture	Fréchet mixture	Lognormal mixture (loc=0)
Global in-sample $R^2$	99.89%	<b>99.98%</b>	99.71%
Global LOO $R^2$	87.44%	<b>95.22%</b>	68.55%
Global in-sample MAE	<b>0.000492</b>	0.001271	0.000824
Global LOO MAE	<b>0.002585</b>	0.004453	0.003137

**Table 3 Fit of different models to observed average water depth quantiles**

How does the CDF differ for different flood zones? It is expected that the  $p_0$  parameter is negatively related to flood zone frequency. The probability of no floods in the lower frequency (e.g., 1-in-100) flood zone is higher than the corresponding probability in the higher frequency (e.g. 1-in-20) flood zone, where floods are more likely. At the same time, the severity of floods, represented by the  $loc$  parameter of the distribution, is expected to be positively related to flood zone frequency (e.g., 1-in-100-year floods cause more damage in the 1-in-20 year flood zone than in the 1-in-100-year flood zone

as houses in 1-in-20 year flood zone are already damaged by 1-in-20 year floods and so 1-in-100-year floods cause even more damage there).

Once the model is calibrated, it enables us to generate Monte Carlo scenarios of yearly future water depths for every house in the dataset. Last step in calculating losses is to transform the simulated water depth into monetary value. This can be done by applying flood depth-damage functions. These are mappings between the flood depth and a percentage reduction in the house price and are based on empirical statistics. Functions for Residential buildings in Europe from (Huizinga et al., 2016) below are used in this study:

Flood depth, [m]	0	0.5	1	1.5	2	3	4	5	6
Damage	0.00	0.25	0.40	0.50	0.60	0.75	0.85	0.95	1.00

**Table 4 Flood depth-damage function**

Note that using numerical method such as Monte Carlo is necessary to value the flood losses in this setup because even though we know the analytical formulas to calculate the expected value of flood water depths given by the Gumbel mixture distribution, by applying nonlinear transformation using flood depth-damage function, we can no longer analytically track what is the resulting expected value after that transformation (due to Jensen’s inequality).

Finally, the loss calculation process under the Monte Carlo simulation is summarized as follows:

1. Simulate percentage loss (flood depth-damage function applied to simulated sample from Gumbel mixture distribution) for one year (or independently for each year if multiple years are used). This study simulates one year loss only. Apply the percentage loss to the house value (in this study assumed to be a constant  $P_0$ ) to get the loss:

$$L_t = perc\_loss_t P_0 \quad (2)$$

2. Sum simulated losses for all the houses in portfolio for the scenario. Then, divide the result by the total value of the portfolio today  $V_0$ . If multiple periods were used, discounting of each loss could be applied to better reflect the time value of money.

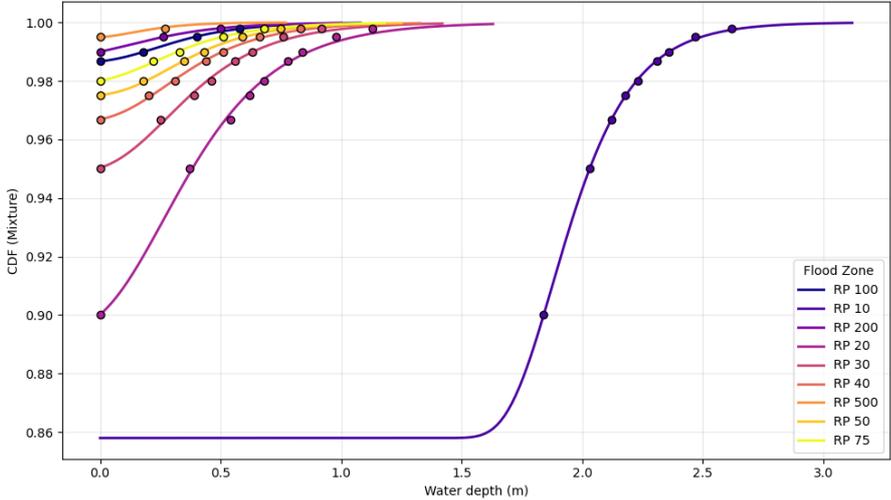
$$Total\ loss\ (\%) = \frac{\sum_{j=1}^{n\ houses} \sum_{t=1}^T L_t}{V_0} \quad (3)$$

### Financial loss model calibration results

We observe only discrete points of CDF for each house, saying the water depth at each return period corresponds to one empirical quantile. Each house is assigned to a flood zone. Empirical quantiles are averaged for all houses within a flood zone. The average quantiles are displayed as points in Figure 1, together with fitted Gumbel mixture CDF. Parameters of the fitted distribution are listed in Table 5.

To interpret the results, let’s simulate a flood event by taking a random draw from continuous standard uniform distribution. Let’s say it is equal to 0.9. The result represents a 1-in-10 year flood event. Now draw a horizontal line at 0.9. The line crosses flood zone FZ 10. In other words, houses in the 10-year flood zone are flooded on average at about 1.8 meters of water. The line also touches the 20-year flood zone at water depth of zero. It means that houses in 20-year flood zone are not flooded yet. Let’s simulate a more extreme flood by drawing 0.96. The horizontal line touches houses in the 10-, 20- and 30-year flood zones. Houses in the 10-year flood zone are hit the most while houses in the 30-year flood zone are hit only slightly. The model behaves as expected, the riskiest areas are hit the most and less risky areas are protected until a more extreme event happens. As the FZ 10 function is an outlier and may distort the results of the analysis, houses located in the zone are removed when the JRC flood

map is used. Furthermore, it is a question whether such houses really exist, e.g. who would like to live in a house that is flooded every 10 years on average? Many of those data points are considered data errors and special cases like houseboats (designed to live on the water). If still these data points would need to be used in any future application or research, suggested approach is to use less outlier estimate of RP 10 using maps from (World Resources Institute, 2020) in Appendix A.



**Figure 1 Calibrated Gumbel mixture CDF**

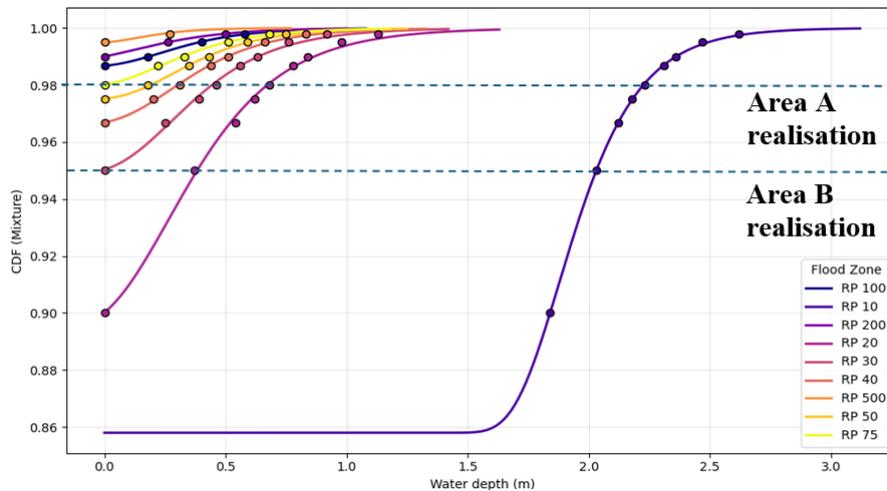
Flood zone	p0	loc	scale
1-in-10	0.857956	1.876300	0.185048
1-in-20	0.893333	0.263168	0.260081
1-in-30	0.948369	0.277348	0.232692
1-in-40	0.965765	0.273125	0.220430
1-in-50	0.974638	0.281502	0.206623
1-in-75	0.978882	0.226617	0.212262
1-in-100	0.986140	0.231646	0.197719
1-in-200	0.987953	0.124831	0.218769
1-in-500	0.994535	0.135007	0.135002

**Table 5 Calibrated Gumbel mixture model parameters**

**Hydrological flow simulation model**

The financial model presented in Figure 1 can generate scenarios of losses given the realization of a standard uniform random variable. For example, if the realization equals 0.95, a 1-in-20 year flood is realized. The model is suitable for pricing flood risk for individual houses. However, on portfolio risk management level, there are multiple houses. Each house is located in a different region with different exposure to flood risk. Flood risk realizations in different parts of the country are not independent. For example, most of the Czech Republic was labeled “in extreme danger” (Czech Hydrometeorological Institute, 2025b) 11<sup>th</sup> September 2024.

To capture this effect in the model, the simulated realizations of the standard uniform random variable should not be independent for different houses. This is demonstrated in Figure 2. Say houses in some area *Area A* experience a 1-in-50 year flood, corresponding to the realization of 0.98. Houses in other part of the country, *Area B*, experience a 1-in-20 year flood, corresponding to the realization of 0.95.



**Figure 2 Dependence of flood return periods for different areas**

To bridge the gap, a hydrological flow simulation model is introduced. The model assigns each house to the nearest hydrometeorological station (*Area*). The station measures average daily water flow in [ $\text{m}^3\text{s}^{-1}$ ]. Next, the maximum daily water flow in a year is assumed to follow the Generalized Extreme Value distribution (limiting distribution for a maximum value out of a block of values). Maximum daily water flows are commonly used in hydrological research (Kulasová, S. et al., 2016) and are used to determine flood stages in flood simulations (Vlasák & Daňhelka, 2020). Generalized extreme value distribution is commonly used in hydrological research (Saksena, 2016).

Using the inverse cumulative distribution function of the fitted distribution, a maximum water flow quantile can be obtained. This quantile can be interpreted as the flood return period experienced in the area. In other words, the inverse CDF of the GEV distribution corresponds to the value that is exceeded with probability  $p=1/T$  where  $T$  is the return period.

Since the hydrological flow model produces return periods, it can be directly linked to the financial loss model introduced earlier. Output of the former model is an input to the latter one. The only remaining question is how to model the return period (i.e. continuous uniform variable realization) dependence among different *Areas*.

Based on the approach how to model dependence of return periods, two methods can be distinguished. First, historical realizations of return periods can be used. These indirectly contain any information about the return period dependence without the need to model it explicitly. The approach is a variant of stress testing when scenarios of past severe events are used to evaluate today's portfolio losses. Second, the dependence can be modeled explicitly using a copula function. The approach can generate any number of future scenarios of return periods, which are then converted to losses. A severe quantile is then used to estimate the extreme, also called tail, portfolio risk. Such quantile is also referred to as value at risk (VaR).

The procedure for stress testing or VaR starts with estimating marginal distributions of maximum water flows. This is done separately for each *Area*, represented by a hydrometeorological station. Next, for each yearly measurement of the maximum water flow, the inverse cumulative distribution function is applied and produces the return period. For example, quantile of 0.95 is produced in the year 2024 and corresponds to the 1-in-20 year flood in the *Area* during that particular year. After that, the two methods differ in how they treat dependence of these return periods. The procedure is discussed in the respective chapters below.

## Stress testing method

The stress testing method includes generating scenarios, both historical or hypothetical (or a combination such as scaled historical scenario) and evaluating the losses on today's portfolio. Since the historical realized return periods for each *Area* are already produced by applying the inverse CDF to the yearly maximum water flow data, the values of quantiles (return periods) can be directly passed to the financial loss model. As a result, for each year for which there is available water flow measurement, a portfolio loss can be estimated. In this study, loss evaluation is done for each year, although only the most severe years are of interest. The purpose is to test the method. If everything is calibrated correctly, the most severe flood years would result in the highest portfolio losses. The most severe flood years in the Czech Republic were 2002 and 1997. Details about Czech historical flood events since 1997 are in the Table 6:

Date	Locality	Cause	Damage (CZK bn)	Infl. Adjusted (2024 prices)
5/1985	Moravia	Extreme rainfall	0.48	3.90
7/1997	Moravia	Extreme rainfall	62.6	153.54
7/1998	East Bohemia	Torrential rain	1.8	3.99
3/2000	North Bohemia, East Bohemia	Heavy rainfall and rapid warming, melting snow	3.8	7.94
8/2002	Bohemia	Torrential rain	73.1	143.24
3/2006	Czech Republic	Heavy warming and rapid warming, melting snow	5.5	9.75
6,7/2009	Czech Republic	Extreme rainfall	5.6	9.25
8,9/2010	North Bohemia, Liberec Region	Extreme rainfall	4	6.51
5,6/2013	Central Bohemia, South Bohemia	Extreme rainfall	15.4	23.48
9/2024	Moravia	Extreme rainfall	45-50	47.50

**Table 6 Overview of historical flood events** (Janasova, 2011);(Seznam News, 2024); (Czech Hydrometeorological Institute, 2013); (Strnadová, 2025); (ČTK, 2009);(Czech Statistical Office, 2025)

There are two types of stress tests with regards to who designs the exercise. First it is the regulatory stress test. For example, the ECB conducted a climate risk stress test of the European banking system in 2022 (Germann et al., 2023). The exercise included a flood risk scenario, which includes severe

floods over a one-year horizon, designed by ECB staff. Second, banks conduct their own internal stress tests, using their own methodology.

### Interaction with credit risk parameters, stress testing LGD

Although banks are not large real estate investors, they are indirectly exposed to the house damage caused by floods, via mortgage collateral. As houses lose value during floods, credit quality of the mortgage deteriorates.

Credit risk is often measured using the expected loss (EL) decomposition:

$$EL_i = PD_i \cdot EAD_i \cdot LGD_i \quad (4)$$

Where  $PD_i$  is the probability of the  $i$ -th borrower default,  $EAD_i$  is the exposure at default and  $LGD_i$  is the loss given default, also decomposed as  $LGD_i = (1 - RR_i)$  where  $RR_i$  is the recovery rate.

Flood risk affects the expected loss via the recovery rate parameter. The more damage is done to the house, the less money is recovered in case the borrower defaults by selling the collateral. Say a percentual damage is applied to the house price in a given scenario via one of the methods presented in this paper (such as VaR or the year 2002 scenario). The stressed  $LGD$  formula is as follows:

$$LGD_{STR,i} = (1 - RR_i(1 - damage\_perc_i)) \quad (5)$$

To analyze the effect of flood risk on portfolio credit risk, further decomposition is used. First, start with the definition of total EL after applying the price shock given floods:

$$EL_{STR,i} = \sum_{i=1}^n PD_i \cdot EAD_i \cdot LGD_{STR,i} = \sum_{i=1}^n EL_i \cdot (1 + ST_i) \quad (6)$$

Where  $EL$  is summed over house collateral  $i=1, 2, \dots, n$  and  $ST_i = \frac{LGD_{STR,i} - LGD_i}{LGD_i}$  is the relative change in LGD for given collateral.

To express the average stress factor for the portfolio, what weights should be used for individual collateral? Assume we don't know yet and label the weights as a vector of unknown variables  $w_i$ . Then, instead of using the  $ST_i$  variable for each of the houses, we would like to use the weighted average stress with unknown weights  $w_i$ .

$$\sum_{i=1}^n EL_i \cdot (1 + ST_i) = \sum_{i=1}^n EL_i \left(1 + \sum_{i=1}^n \frac{w_i}{\sum_{i=1}^n w_i} ST_i\right) \quad (7)$$

As the weighted average stress is a constant, it can be taken out of the sum:

$$\sum_{i=1}^n EL_i \cdot (1 + ST_i) = \left(1 + \sum_{i=1}^n \frac{w_i}{\sum_{i=1}^n w_i} ST_i\right) \sum_{i=1}^n EL_i \quad (8)$$

This can be further expanded into:

$$\sum_{i=1}^n \frac{w_i}{\sum_{i=1}^n w_i} ST_i = \frac{\sum_{i=1}^n EL_i \cdot (1 + ST_i)}{\sum_{i=1}^n EL_i} - 1 = \frac{\sum_{i=1}^n EL_i \cdot ST_i}{\sum_{i=1}^n EL_i} \quad (9)$$

The weight was found to be  $w_i = EL_i$ .

As a result, the individual values of stressed LGD can be forgotten, what matters is the EL weighted percentage LGD stress on the portfolio level. This single number should be the output of a stress test exercise.

### Value at Risk method

The VaR method presented below uses the Copula function to explicitly model the dependence of return periods in different *Areas*. Copula is a function that connects marginal distributions to form a

joint distribution. For a bivariate joint cumulative distribution function  $F_{X,Y}(x,y)$  with marginals  $F_X(x)$  and  $F_Y(y)$ , there exists a copula  $C(u,v)$  such that:

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y)) \quad (10)$$

The copula function captures all the dependency structure between two or more random variables, in this example  $X$  and  $Y$ . Marginal distributions are separated out. Furthermore, the copula density  $f_{X,Y}(x,y)$  can be represented as a product of 3 factors, marginal densities  $f_X(x)$ ,  $f_Y(y)$  and copula density  $c(F_X(x), F_Y(y))$ .

$$f_{X,Y}(x,y) = c(F_X(x), F_Y(y))f_X(x) f_Y(y) \quad (11)$$

The intuition is that the product of marginal densities  $f_X(x)$ ,  $f_Y(y)$  would be the joint density if  $X$  and  $Y$  were independent and the copula density is the adjustment factor, capturing dependence between  $X$  and  $Y$ .

Let  $U = F_X(X)$  and  $V = F_Y(Y)$ . Since a random variable is passed into the CDF, a random variable is also returned.  $U$  and  $V$  are standard uniform random variables. In this study, it is the variable determining flood return periods for two different *Areas*. The dependency structure between the return periods is driven by the copula function.

A special type of copulas, Archimedean copulas, are of special importance in this study. The general representation of an Archimedean copula is:

$$C(\mathbf{u}) = \varphi_0(\varphi_0^{-1}(u_1) + \dots + \varphi_0^{-1}(u_d)), \mathbf{u} \in [0,1]^d \quad (12)$$

Here,  $\varphi$  is called the generator of Archimedean Copula. It is a nonincreasing function, mapping  $[0, \infty]$  to  $[0, 1]^d$ . However, some sources rather present the inverse of the function so that the function and its inverse notation is swapped (Shaw, 2011). Chosen generator determines type of tail behavior. Since in this study, water flows are measured, right tail dependence is assumed (high water flows occur together more likely). For this reason, Gumbel Copula is chosen. Gumbel copula exhibits strong upper tail dependence and no lower tail dependence, as can be seen in Figure 3. The generator function is:

$$\varphi(t) = \exp(-t^{\frac{1}{\theta}}) \quad (13)$$

The Gumbel copula is described below:

$$C(\mathbf{u}) = \exp([(-\ln u_1)^\theta + \dots + (-\ln u_d)^\theta]^{\frac{1}{\theta}}) \quad (14)$$

The  $\theta \geq 0$  parameter controls the strength of upper tail dependence. The higher the parameter, the stronger the dependence (extreme points of  $u$  and  $v$  near  $[1,1]$  occur together). The theta parameter is related to the Kendall's tau correlation by:

$$\theta = \frac{1}{1 - \tau} \quad (15)$$

If more than two variables (like  $u_1$  and  $u_2$ ) are involved, Kendall's Tau for all the pairs of random variables is averaged and then converted into an estimate of  $\theta$ . In this study, the variables ( $u_1$ ,  $u_2$  etc.) are quantiles obtained via inverse CDF of GEV distribution of the maximum yearly water flows, representing return periods for different hydrometeorological stations.

After Gumbel copula is calibrated to the data using the Kendall's tau, the next step is to conduct a simulation (Shaw, 2011); (Hofert, 2007). In this study, the Marshall Olkin (Marshall & Olkin, 1988) algorithm is used:

1. Sample  $V_0 \sim F_0 = \mathcal{L}S^{-1}(\varphi_0)$
2. Sample i.i.d.  $X_i \sim U[0,1], i \in \{1, \dots, d\}$
3. Return  $(U_{1, \dots, U_d})$  where  $U_i = \varphi_0(-\log(X_i/V_0)) i \in \{1, \dots, d\}$

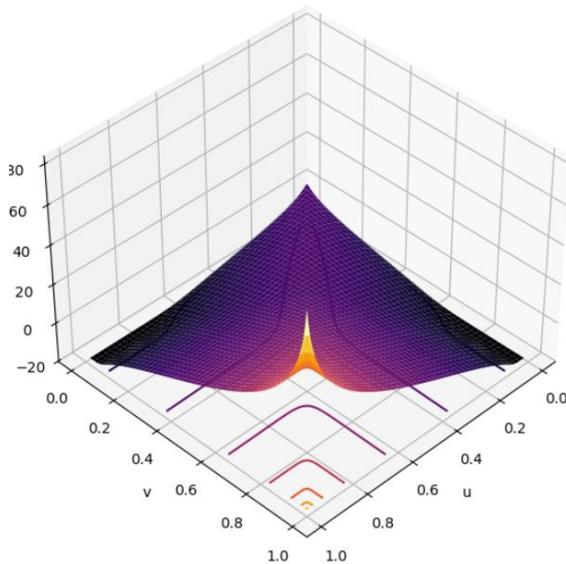
Where  $\mathcal{L}S^{-1}(\varphi_0)$  denotes the inverse Laplace-Stieltjes transform of  $\varphi_0$ . Only one sample  $V_0 \sim F_0$  is required, independent of the dimension, plus additional d random numbers  $X_i$ , in total  $d+1$  random numbers are required.

The Laplace-Stieltjes transform of  $\varphi_0$  for the Gumbel copula is (Hofert, 2007):

$$F_0 = S\left(\frac{1}{\theta}, 1, \cos\left(\frac{\pi}{2\theta}\right)^\theta, 0; 1\right) \quad (16)$$

Where S is the Stable distribution  $S(\alpha, \beta, \gamma, \delta; 1)$  with exponent  $\alpha \in (0, 2]$ , skewness parameter  $\beta \in [-1, 1]$ , scale parameter  $\gamma \in [0, \infty)$ , and location parameter  $\delta \in \mathbb{R}$ .

The final result, simulated  $U_{1, \dots, U_d}$  sample of dependent uniform random variables, represents the realization of flood return periods (Figure 2) for different Areas in the country.



**Figure 3 Simulated Gumbel Copula density**

### Copula goodness of fit

To determine if the Copula choice was right, the goodness of fit of the empirical copula versus the model copula is checked. This is done using the Cramér–von Mises test criterion (Anderson, 1962). The null hypothesis says that the data  $[0, 1]^d$  was generated by Gumbel copula( $\theta$ ) for some value of unknown parameter  $\theta$ . The alternative hypothesis says that the true copula is not the Gumbel copula. In this article, data is realized quantiles (return periods) of water flows. In general, it is not the original dataset, but a dataset converted to quantiles (here using the GEV inverse CDF).

Given n observations  $u_i = (U_{i,1}, \dots, U_{i,d})$  in d-dimensions, the empirical copula distribution function given by:

$$C_n(\mathbf{u}) = \frac{1}{n} \sum_{i=1}^n I(U_{i1} < u_1, \dots, U_{id} < u_d), \mathbf{u} \in [0, 1]^d \quad (17)$$

Where  $I(\cdot)$  denotes the indicator function, equal to 1 if the condition holds and 0 otherwise.

The Cramér–von Mises test statistic is given by (Genest & Rémillard, 2008); (Hodel, 2025):

$$T = \sum_{i=1}^n (C_n(\mathbf{u}_i) - C_\theta(\mathbf{u}_i))^2 \quad (18)$$

Where  $C_\theta$  is the theoretical copula distribution function with  $\theta$  parameter under the  $H_0$ .

In practice, the distribution of  $T$  under the  $H_0$  is calculated using numerical procedures such as the one-level parametric bootstrap (Genest & Rémillard, 2008) by simulating a large number of samples from the theoretical copula. For each sample, the  $T$  statistic is calculated using the sample estimate of  $\theta$ . Then, the  $T$  statistic on the data sample is compared to this simulated distribution. If it lies in the upper tail,  $H_0$  is rejected (we are concerned about the right tail as the test statistic measures squared errors so that the higher it is, the bigger the error). The p-value is the upper-tail exceedance frequency.

### III. Data and Variable Measurement

#### Data

All the data regarding house attributes has been given to me by *Seznam.cz* for the purpose of this article. *Seznam.cz* is one of the largest and most influential reality portals in the Czech Republic. The data cover 40 different parameters of 236 226 house advertisements over the period 12/2023 – 12/2024, including the bid price of the house and location information (district, geographic coordinates, address). It is assumed that this sample represents a hypothetical house or house collateral portfolio of an institution and prices of houses remain constant within the simulated period of 1 year. Flood risk of the house portfolio is calculated in this study.

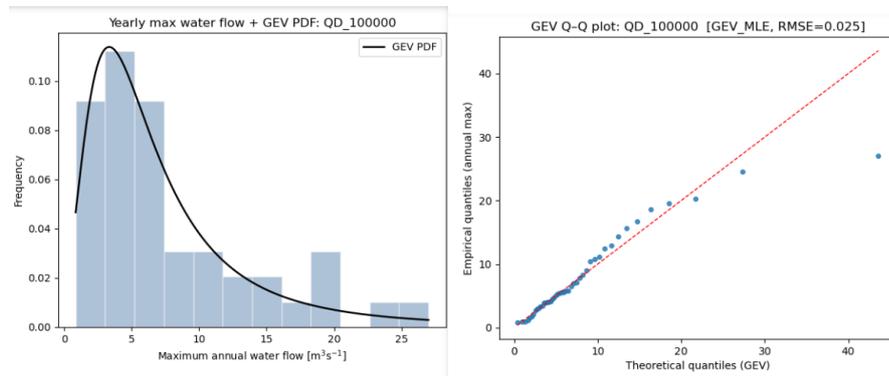
The data required a high degree of data cleaning as it is data entered by customers on the website, subject to error or manipulation. First, houses located outside the Czech borders or located inside rivers were removed. Next, houses with suspiciously extreme prices (under 300 000 CZK and over 300 000 000 CZK) were removed. Houses with extreme estate area (above 100 000 m<sup>2</sup>) were removed. Houses with insufficient address information were removed (minimum requirement is street name and for towns with less than 1 000 inhabitants, the minimum is municipality name and house number). Finally, only last advertisement for each house was kept as there were many repeated advertisements for the same house (on average 4).

Regarding the flood maps, two sources were used. First, the JRC (European Commission, JRC, 2025) river flood hazard dataset and second the VUV (T. G. Masaryk Water Research Institute, 2025) flood zone map. While the former source has more details – it maps every point in the country directly to water depth at given return period and covers a broader range of return periods (10, 20, 30, 40, 50, 75, 100, 200 and 500), the latter is an official, local source whose responsibilities are governed by (Act on Waters and Amendments to Certain Acts (Water Act), 2001). However, it only maps every point in the country to flood zone and is available only for the 20, 100 and 500 return frequency. As a data error was encountered in the 500 years map, only 20 and 100 maps were used in this study, although interpolation of maps following (Zarekarizi et al., 2021) was attempted with limited success (interpolation error was considered too big). The study makes use of both sources. The JRC map is used to calibrate the Gumbel mixture model. Then, the calibrated model is used separately for JRC flood zones and the local flood zones to produce alternative measures of flood risk.

For the hydrological flow model, daily water flow data for hydrometeorological stations in Czech Republic from (Czech Hydrometeorological Institute, 2025a) are used. Since some stations have a longer data history than others, only stations with data coverage 1980-2024 are kept. Finally, 336 stations are used. Then, each station is geolocated using Google maps (Google, 2025). Each house in the dataset is assigned to the nearest hydrometeorological station.

### Hydrological flow model calibration results

Data on average daily water flow for different hydrometeorological stations are aggregated into yearly maximum values. Then, for each station, generalized extreme value distribution is fitted on the yearly maximum water flows. In Figure 4, the fitted distribution for a hydrometeorological station with identifier QD\_100000 is plotted together with the histogram of yearly maximum water flows. Next to the graph, the quantile-quantile plot is used for the diagnosis of fit quality. Generally, the GEV distribution fits the data well as the points in the Q-Q plot are along the 45 degree line.



**Figure 4 Generalized extreme value distribution fit**

Next, each yearly maximum value is converted into a quantile of the fitted GEV distribution using the inverse cumulative distribution function of the GEV distribution. The obtained indicate the flood return period for each combination of hydrometeorological station and year.

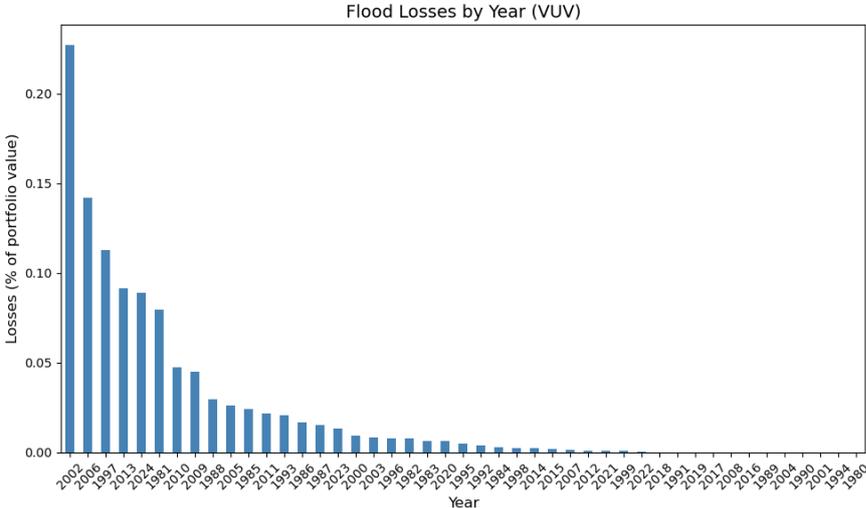
## IV. Empirical Results

### Stress testing

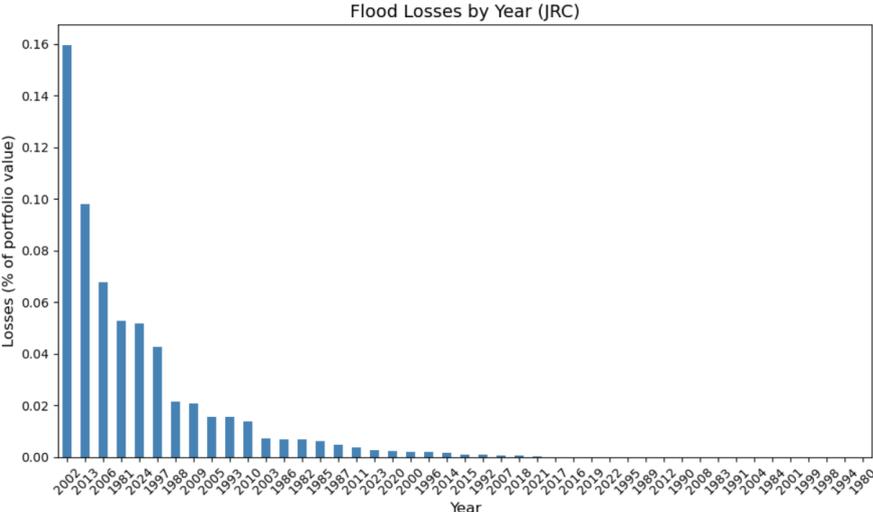
The stress testing exercise uses the historical quantiles of GEV distribution obtained in the previous step. These quantiles (flood return periods) are assigned to each house based on the nearest hydrometeorological station. Say the realized quantile for a particular house is equal to 0.95. In other words, the house (and surrounding houses in the proximity of the same hydrometeorological station) experience a 1-in-20 year flood. After that, the financial loss model uses the 0.95 quantile to generate financial losses for the house. The losses for each house are summed up and divided by the total value of the portfolio (sum of the house prices). The exercise is repeated for each year for which the data on water flow is available (1980-2024). In addition, the exercise is done separately using VUV maps and JRC maps. Resulting distribution of losses is captured in Figure 5 and Figure 6.

If the stress testing exercise was done correctly, the resulting losses would copy the historical record of flood event severity in Table 6. Generally, the results are in line. The most extreme losses in both simulations are in the year 2002, which is in line with historical experience in Table 6. Also, other extreme floods in the year 1997, 2014 and 2024 are among the highest simulated losses. The comparison with historical records proves that the stress testing methodology developed earlier is producing results that make sense and are expected.

The stress testing methodology could be easily extended by further stressing the historical experience. For example, realized quantiles of GEV distribution could be shifted such that houses that experienced a 1-in-100 year flood would now experience a 1-in-500 year flood. The resulting scaled scenario would preserve the historical dependence between the *Areas* while it would allow us to simulate more extreme floods than were experienced before. Moreover, flood maps under different climatic scenarios could be used to further enlarge the extent of flood Zones. Flood hazard maps under future climate scenarios are discussed in (Darlington et al., 2024).



**Figure 5 Simulated flood losses using the VUV flood map (Stress test)**



**Figure 6 Simulated flood losses using the JRC flood map (Stress test)**

**Backtest**

As a simple backtest, damage estimate by the model was compared to the actual inflation adjusted historical flood losses estimates in Table 6 for 10 years of largest floods in the 1980-2024 period. Since the model measures risk in different units, the losses were converted to ranks so that the worst flood year among the 10 worst is labelled 1 and the year with the minimum loss is labelled 10. Loss rank results are displayed in Table 7. The model outputs are very close to reality. For example, the year of 2002 is the second worst according to inflation adjusted loss and the worst by both models. The degree of rank correlation is measured by Spearman’s correlation coefficient in Table 8 and is about 80

percent for both models. To conclude, both models return similar flood severity as it was observed in history.

Date	ST loss JRC - rank	ST loss VUV - rank	Infl. Adjusted (2024 prices) - rank
5/1985	8	8	10
7/1997	5	3	1
7/1998	10	10	9
3/2000	9	9	7
8/2002	1	1	2
3/2006	3	2	5
6,7/2009	6	7	6
8,9/2010	7	6	8
5,6/2013	2	4	4
9/2024	4	5	3

**Table 7 Model results versus historical damage**

$\rho$ (model <sub>JRC</sub> , infl. adjusted damage)	$\rho$ (model <sub>VUV</sub> , infl. adjusted damage)
0.782	0.806

**Table 8 Spearman correlation estimate**

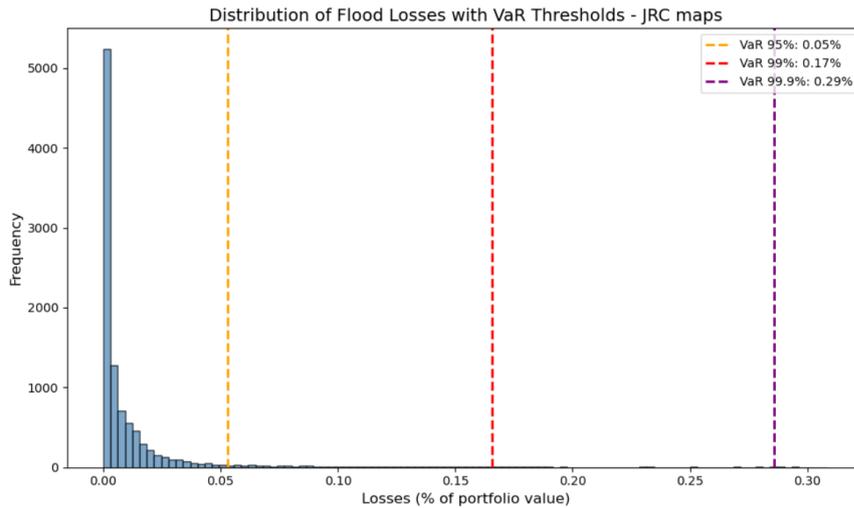
**Value at Risk**

The Value at Risk exercise applies the Gumbel copula method to historical quantiles of GEV distribution obtained in the previous step. First, the average Kendall’s tau between pairs of these quantiles is estimated, together with a bootstrap confidence interval constructed by resampling yearly measurements of the realized quantiles with repetition, without re-estimating the marginal distributions at each step (this was done to save computational time and may lead to slight underestimating). The resampling is done 300 times and the 90 percent confidence interval around the Kendall’s tau is estimated. The results are:

$\tau_{0.05}$	$\tau$ estimate	$\tau_{0.95}$
0.220	0.287	0.344

**Table 9 Kendall’s tau estimate**

The Value at Risk exercise simulates losses over 1 year by first drawing dependent uniform variables for each hydrometeorological station using the Marshall Olkin algorithm discussed earlier in II, which is then used for each house in its neighbourhood and passed into the financial loss simulation model which generates losses. Losses are aggregated for all houses in the portfolio and then, the total value of losses is divided by the total portfolio value (sum of house prices). The exercise is done separately for VUV and JRC flood maps for different quantiles of VaR and the baseline estimate of the tau parameter (determining the copula theta parameter). Number of Mone Carlo simulation of losses is set to 10 000. The reason is to be able to calculate quantiles with higher precision, such as 99.9 (1-in-1000 year event).



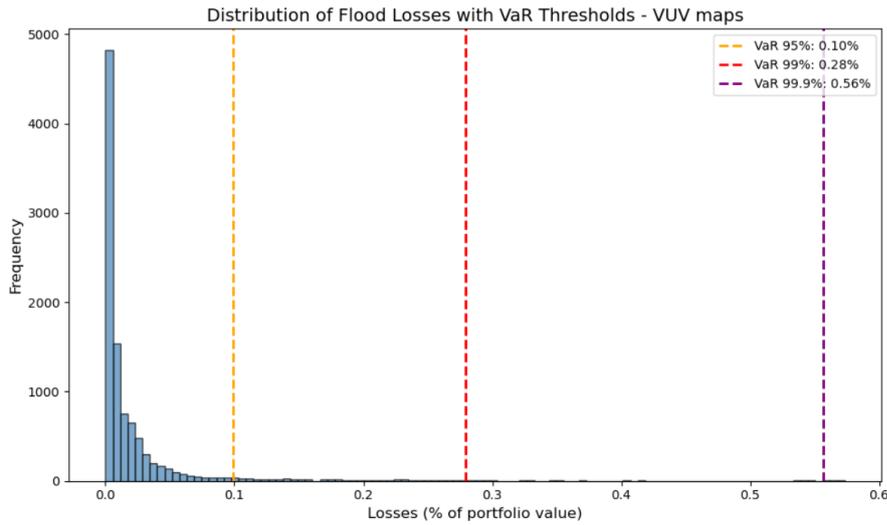
**Figure 7 Simulated flood losses using the JRC flood map (VaR)**

Next, the results are compared to results under the independence assumption, perfect dependence and stressed values of the tau parameter. In Table 10, the VaR results are reported under different assumptions. First, independence between return periods at each place in the country is assumed. The independence assumption results in the smallest risk estimates. Although unrealistic, it gives us a lower bound estimate of the risk. Next, the Gumbel Copula is run with baseline and extreme but plausible values of tau from Table 9. The higher the tau, the higher the tail dependence and the higher the resulting risk estimate is. On the other end, perfect dependence between places on the map is assumed. This assumption would correspond to unrealistic situation where all the places in the country experience the same return period floods each year and gives an upper limit of the risk estimate. Note that the 99.9% are very stable across assumptions. It is therefore a good measure of tail risk for floods.

VaR level	Portfolio loss (in % of total portfolio value)				
	Independence	$\tau_{0.05}$	$\tau$ estimate	$\tau_{0.95}$	Perfect dependence
95%	0.04%	0.05%	0.05%	0.06%	0.10%
99%	0.05%	0.15%	0.17%	0.18%	0.25%
99.9%	0.06%	0.28%	0.29%	0.29%	0.29%

**Table 10 Measurement risk assesment using the JRC flood map (VaR)**

The results using VUV maps are captured in Figure 8 and Table 11 below, while the same conclusions apply:



**Figure 8 Simulated flood losses using the VUV flood map (VaR)**

VaR level	Portfolio loss (in % of total portfolio value)				Perfect dependence
	Independence	$\tau_{0.05}$	$\tau$ estimate	$\tau_{0.95}$	
95%	0.06%	0.09%	0.10%	0.11%	0.17%
99%	0.08%	0.25%	0.28%	0.29%	0.55%
99.9%	0.10%	0.56%	0.56%	0.56%	0.57%

**Table 11 Measurement risk assesment using the VUV flood map (VaR)**

### Copula goodness of fit test

The assumption of Gumbel copula was tested using the Cramér–von Mises test statistic in Table 12. The null hypothesis states that the data (flood return periods) were generated by the specified copula. According to the results, Gumbel and Clayton copulas are suitable while the test is most confident in Gumbel copula. Gauss copula is rejected.

Copula	Gauss	Gumbel	Clayton
CvM test p-value	0.049%	55.32%	39.37%
Test result (5% significance level)	rejection	No rejection	No rejection

**Table 12 Copula goodness of fit test**

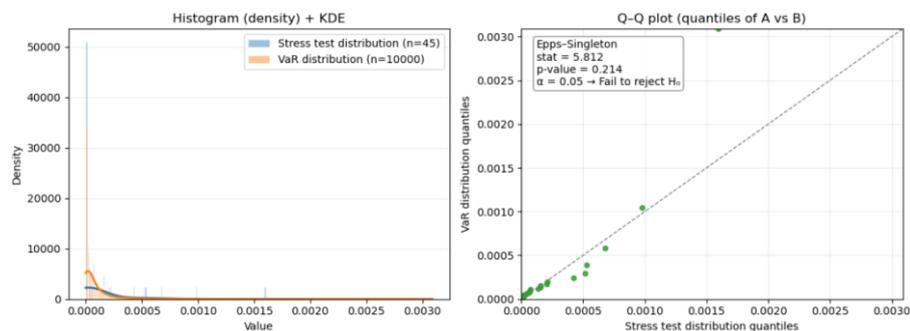
The results confirm the assumption that extreme flood events tend to occur at many places at the same time as was demonstrated in Table 6 or the labelling of the whole country in extreme danger in (Czech Hydrometeorological Institute, 2025b). On the other end of the spectrum, extreme drought can also be correlated across regions. However, the model produces losses only on extreme flood events. On extreme drought events, the damage is zero. Therefore, focusing the model to capture extreme flood events is more important and Gumbel copula is ideal for that purpose. Second, fitting Gumbel copula is much simpler than fitting elliptical copulas on the flood return period data as it is high dimensional (45 years x 336 stations). Estimating a correlation matrix on high dimensional data is difficult as it needs to be invertible to calculate CDF later for the Cramér–von Mises statistic. The

correlation matrix in Table 12 for Gaussian copula was produced using the (Ledoit & Wolf, 2004) shrinkage estimator. On the other hand, Gumbel copula requires only a single dependence parameter, linked to the Kendall's tau. It turns out the using a single parameter for the whole Czech Republic is suitable. For larger countries, dividing hydrometeorological stations into clusters based on river basins and using a vine copula structure could be a solution.

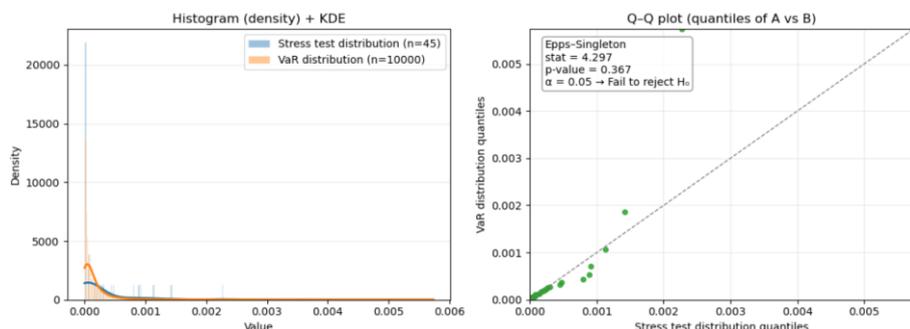
To conclude, Gumbel copula is preferred first because it fits the data best according to Cramér–von Mises goodness-of-fit test. Second, the focus of the model is to capture extreme flood events (right tail) which is modeled well by Gumbel copula. Third, Gumbel copula is easy to estimate as it uses a single dependence parameter as opposed to elliptical copulas which are computationally heavy and estimates can be biased and uncertain in high dimensions.

### Stress test model versus VaR loss distribution test

Another diagnosis check was done by comparing the resulted loss distribution of both exercises, stress test (characterized by observed dependence structure) and VaR (Gumbel copula dependence structure). In Figure 9 and Figure 10, the histogram with kernel density estimate is compared for both loss distributions in the left picture and a q-q plot with Epps-Singleton test result is captured in the right picture. For both flood map sources, the null hypothesis that the data come from the same distribution is not rejected. The result supports the hypothesis that the Gumbel copula model captures dependence structure well and it demonstrates the integrity of both exercises. Moreover, both exercises could be connected as the VaR model is able to assign probability to the stress test generated losses.



**Figure 9 Stress test versus VaR loss distribution test JRC flood map**



**Figure 10 Stress test versus VaR loss distribution test VUV flood map**

## V. Discussion

The study shows that the proposed hydrological flow model, combined with financial loss model generates sensible results. Especially, it can replicate the order of historical flood magnitudes, presented in Table 6. Moreover, the VaR method is able to simulate events not experienced in recent history.

Although statistical assumptions of the model were tested against reality, the biggest model uncertainty comes from data quality and aggregation. Some houses' addresses were not complete and either must have been discarded or assigned to unprecise location (the middle of the street). Financial loss model was estimated on aggregated data (flood zones) instead of each individual house separately. The reasons were to reflect that some house locations are not perfect, to eliminate possible hydrological model outliers (as it turned out, flood maps differ across different providers) and to simplify the model (it would be hard to demonstrate and diagnose 30 000 estimated CDFs). Only flood zones with return period 20 years or higher were considered as lower return periods would cause large outliers and could amplify imprecise house location mapping. Another possible shortcoming of the method is that it is a single factor model as the only parameter, Kendall's tau, determines the dependence structure among all the hydrometeorological stations in the country. In reality, the dependence is higher for regions closer to one another than for regions farther away from each other. However, the simplification is plausible as the Czech Republic is a small country and flood events often encompass a large part of the country. For bigger countries, possibly a vine copula structure could allow the parameters to vary across different regions.

## VI. Further research

The proposed method could be used in combination with flood hazard maps for different future climate scenarios, such as optimistic/pessimistic Aqueduct Floods maps for horizon 2030 or 2050 provided by (World Resources Institute, 2020).

## VII. Conclusion

The study established a theoretical framework for measuring flood risk of real estate portfolios or real estate collateral. The advantage of the proposed methodology is the ability to replicate the order of historical flood events severity and generate future scenarios beyond those observed in the past. The study introduced elements which are not present in many stress tests or financial stability exercises, namely the introduction of continuous marginal distributions of flood losses on an asset level together with the dependence structure (i.e., diversification) of these flood losses. Such framework corresponds to the traditional financial risk measurement approaches. Using such method, financial institutions and regulators can operate with a practical flood risk measure based on realistic assumptions, comparable across different institutions and free of ad hoc assumptions for specific one-time purpose. The method could be applied as a part of the climatic stress test or capital requirement calculation in banking as well as an assessment of flood risk for large institutional holders of real estate or as a tool to forecast future flood losses for insurance companies or state institutions.

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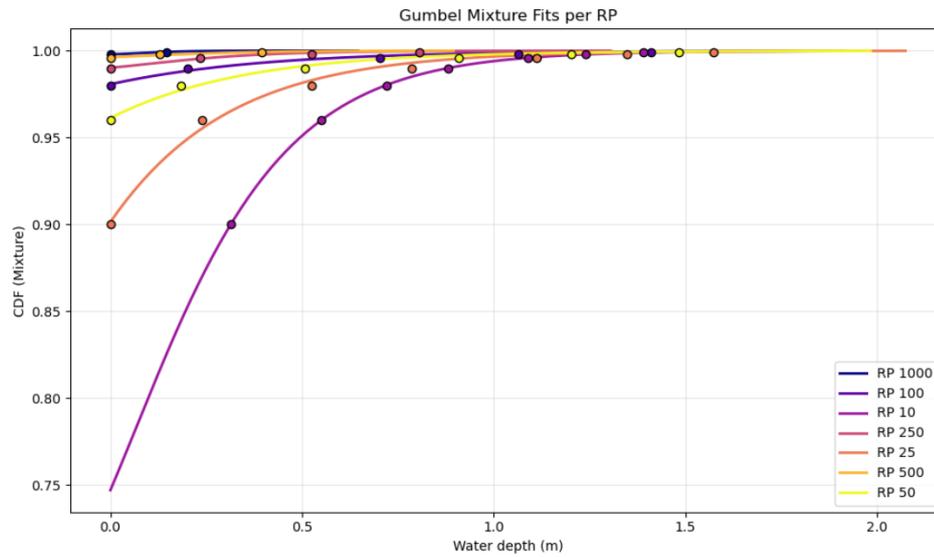
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## Appendix

### A. Alternative Gumbel mixture CDF estimate using (World Resources Institute, 2020)



**Figure 11**

Flood zone	p0	loc	scale
1-in-10	0.746389	0.067191	0.23098
1-in-25	0.902053	0	0.231718
1-in-50	0.961724	0	0.260666
1-in-100	0.980993	0	0.297793
1-in-250	0.990001	0.010418	0.232661
1-in-500	0.996335	0.125065	0.15959
1-in-1000	0.997886	0.068664	0.066043

**Table 13 Note:** loc param was restricted on non-negativity

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